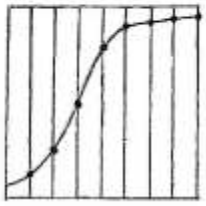
Section 5.7

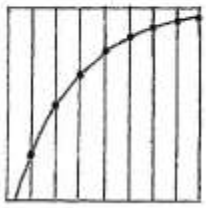
 # 131

Section 5.7

(131) Decide whether the exponential, logarithmic or logistic model best describes the following fitted scatterplots.

(a) 

(b) 

(c) 

Part-a

We can either use an exponential function or logarithmic function and calculate the R^2 value.
We choose the model with the higher R^2

Part-b

We can choose a "logistic model" because the y-value approaches a certain value as x becomes large and as x goes to the left the y value also approaches a certain value.

Part-c

We can either use an exponential function or logarithmic function and calculate the R^2 value.
We choose the model with the higher R^2

 # 132

(132) Make a scatterplot of the data. Then find an exponential, logarithmic or logistic function f that best models the data. (Hint: Use R^2 when deciding between the exponential and logarithmic models.)

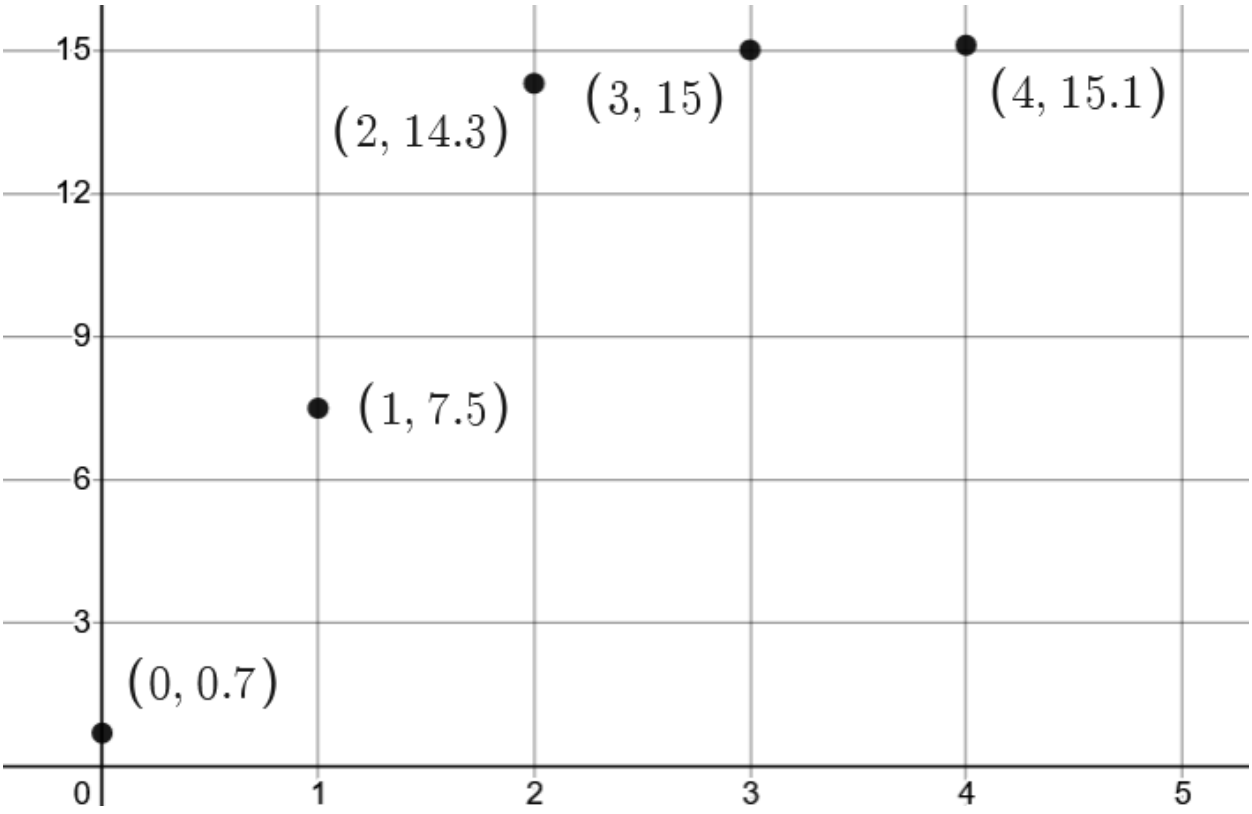
(a)

x	0	1	2	3	4
y	0.7	7.5	14.3	15	15.1

(b)

x	1	2	3	4	5
y	9	5.5	3.5	2.1	1

Part-a



One of the x-value is 0 so in order to use the logarithmic function, we first add x-values by 1.

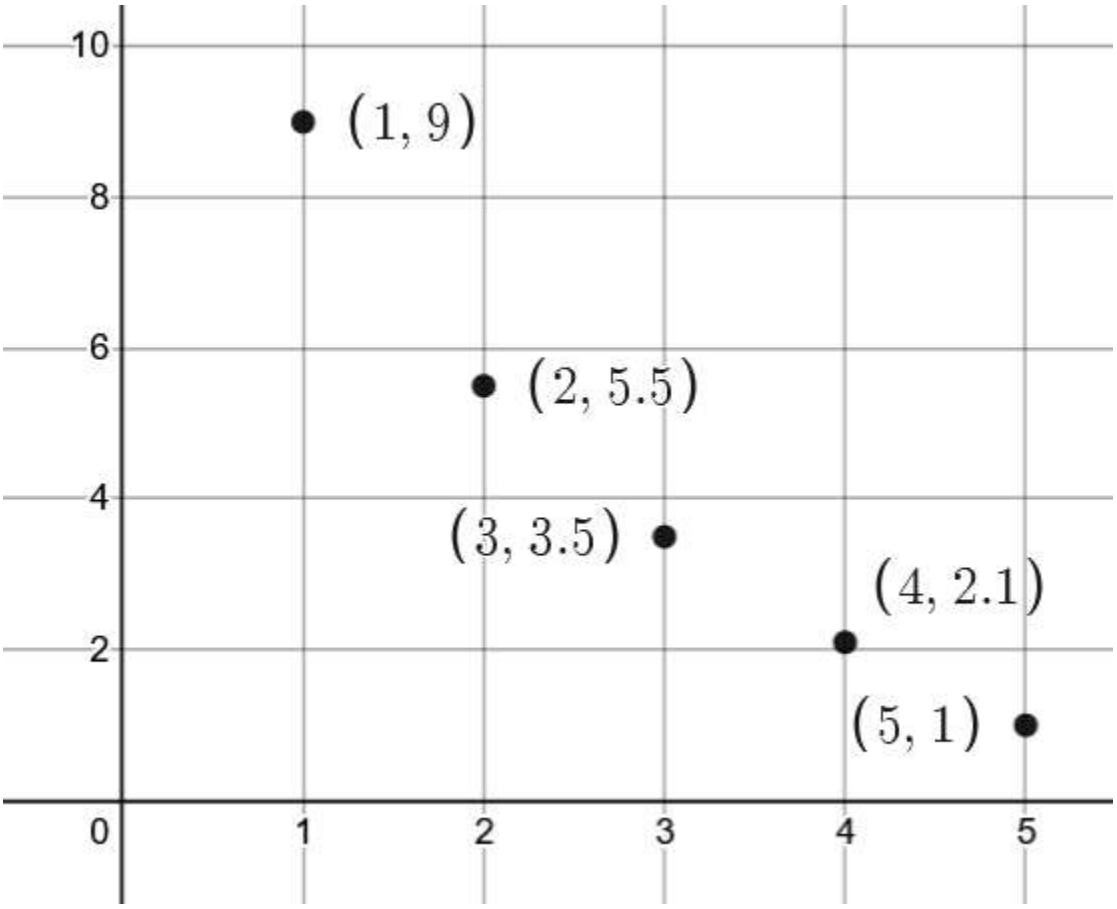
According to the calculator, the best logarithmic function for the data is

$f(x^*) = 9.6821\ (\ln x^*) + 1.2494$ with an R^2 of 0.9413 where x* = x+1

In terms of x, the model is $f(x) = 9.6821\ (\ln (x + 1)) + 1.2494$

We did not choose the exponential model because according to the calculator it has an R^2 of 0.4945 verses the R^2 of 0.9413 for the logarithmic model

Part-b



According to the calculator, the best logarithmic model for the data is
$f(x) = -4.968\ (\ln x)\ +\ 8.9765$ with an R^2 of 0.9999.
On the other hand, the best exponential model is $f(x) = 16.224e^{-0.536x}$ with an R^2 of 0.9962
On the basis of R^2, we choose the logarithmic model $f(x) = -4.968\ (\ln x) + 8.9765$

 # 133

(133) In a town whose population is 3500, a disease creates an epidemic. The number of people N infected t days after the disease has begun is given by the function: $N(t) = \dfrac{3500}{1 + 19.9\,e^{-0.6t}}$.

(a) How many are initially infected with the disease ($t = 0$)?

Part-a

$$N(0) = \frac{3500}{1+19.9e^{-0.6(0)}} = \frac{3500}{20.9} \approx 167$$ (rounded to the nearest whole number)

Initially, 167 people are infected with the disease

Part-b

(b) Find the number of infected after 2 days and 7 days.

$$N(2) = \frac{3500}{1+19.9e^{-0.6(2)}} = \frac{3500}{6.9938} \approx 500$$ (rounded to the nearest whole number)

$$N(7) = \frac{3500}{1+19.9e^{-0.6(7)}} = \frac{3500}{1.2984} \approx 2696$$ (rounded to the nearest whole number)

After 2 days, approximately 500 will be infected.
After 7 days, approximately 2696 will be infected.

Part-c

(c) When will 90% of the population be infected ?

We want to solve for t so that $N(t) = 0.9(3500)$

$$\frac{3500}{1 + 19.9e^{-0.6t}} = 0.9(3500)$$

$$1 + 19.9e^{-0.6t} = \frac{10}{9}$$

$$19.9e^{-0.6t} = \frac{1}{9}$$

$$e^{-0.6t} = \frac{1}{9(19.9)}$$

$$-0.6\,t = \ln\left(\frac{1}{9(19.9)}\right)$$

$$-0.6\,t = -\ln(179.1)$$

$$t = \frac{\ln(179.1)}{0.6} \approx 8.65 \text{ days}$$

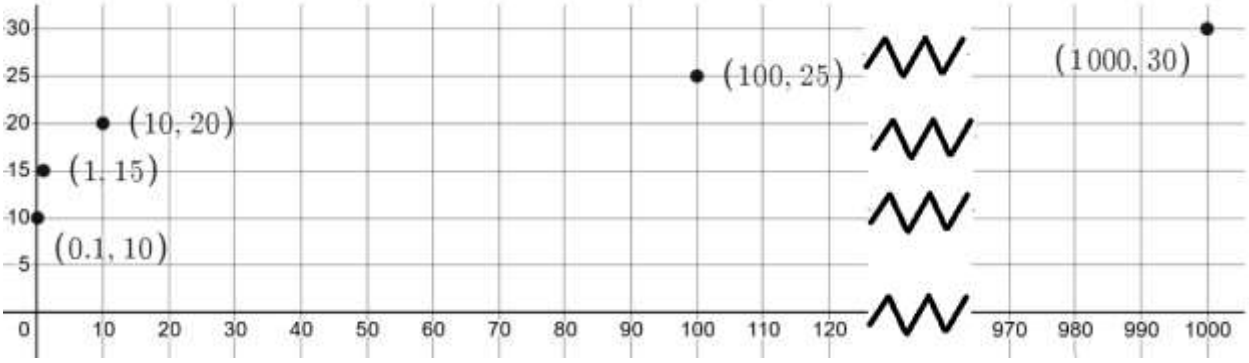
90% of the people will be infected after **8.65 days**

 # 134

(134) Near New Guinea there is a relationship between the number of bird species found on an island and the size of the island. The table lists the number of species of birds y found on an island with an area of x square kilometers.

x (km^2)	0.1	1	10	100	1000
y (species)	10	15	20	25	30

(a) Find a function f that models the data. Write it down.



According to the scatter plot above, the logarithmic model best describes the data.
According to the calculator, $f(x) = 2.1715 \ln(x) + 15$

Part-b

(b) Predict the number of bird species on an island of 5000 km^2.

If x = 5000, then
$f(5000) = 2.1715 \ln(5000) + 15 \approx 33.495 \approx 33$ (rounded to the nearest whole number)
We predict that the number of bird species is 33

Part-c

(c) If there are 50 bird species on an island, how many square kilometers would you expect the island to be?

We solve for "x" so that f(x) = 50

$2.1715 \ln(x) + 15 = 50$

$2.1715 \ln(x) = 35$

$\ln(x) = 35/2.1715$

$x = e^{35/2.1715} \approx$ **10 million square km**

 # 135

(135) The Center for Unexplained Phenomena tracks UFO sightings worldwide. The following table represents sightings for selected years since 1950.

x	1950	1965	1980	1995	2010
f(x)	2500	833	270	93	31

(a) Find an exponential or logarithmic function f that models the data, where x = 1 corresponds to 1950, x = 2 corresponds to 1965, and so on. Justify your answer. Round to 2 decimal places.

Part-a

According to the calculator, the best logarithmic model for the data is
$f(x) = -1557 \ln x + 2236.5$ with an R^2 of 0.9222.
On the other hand, the best exponential model is $f(x) = 7439.68e^{-1.097x}$ with an R^2 of 1
On the basis of R^2, we choose the exponential model $f(x) = 7439.68e^{-1.097x}$

Part-b

(b) During what year did sightings total 100?

We solve for x so that f(x) = 100

$7439.68e^{-1.097x} = 100$

$e^{-1.097x} = 100/7439.68$

$-1.097x = \ln(100/7439.68)$

$x = \frac{\ln(100/7439.68)}{-1.097} \approx 4$ which corresponds to **Year 1995**

Part-c

(c) Extrapolate sightings in 2025.

The year 2025 corresponds to x = 6

If x= 6, then

$f(6) = 7439.68e^{-1.097(6)} \approx 10.30 \approx 10$